DECEMBER 6, 1995 WOCOMAL VARSITY MEET

ROUND I: Arithmetic — Percent, interest, discount, fractions & decimals

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. A sports team has won 30 of its first 40 games. What is the smallest number of the remaining 42 games that the team must win to win at least 84% of all its games?

2. Find the difference, as a reduced fraction, between the largest and second largest of the following:

$$\frac{1}{2}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}$$

3. The treasurer of a Health Maintenance Organization (HMO) with a total revenue of 2 million dollars has advised that if the HMO increases the amount that they charge for a prescription from \$4 to \$7, the HMO will increase its total revenue by 12%. What percent of the 2 million dollars is from selling prescriptions?

ANSWERS

(1 pt) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Auburn, Leicester, Notre Dame

ROUND II: Algebra 1 — open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. In triangle ABC, $\angle A$ is twice $\angle B$ and $\angle B$ is five more than one-half of $\angle C$. Find $\angle B$ in degrees.

2. A bank has two types of checking accounts. The first charges \$5 per month and 2¢ per check. The second charges \$2 per month and 8¢ per check. At most how many checks per month can a holder of the second type of account write if he wants to keep his cost less than it would be with the first type?

3. If $x^7 - 41x^5 + 80x^3 - 127x^2 + 78x - 27 = (x^3 - 7x^2 + 5x - 3) \cdot P(x)$, find the numerical coefficient of the x-term of the polynomial $P(x)$.

ANSWERS

(1 pt) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Auburn, Doherty, Westboro

ROUND III: Set theory and logic problems

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. Set B has 12 more subsets than set A. How many elements are there in set A?

2. We can write dates like December 31, 1995 as 12 / 31 / 95 and we can say that this date has a date-sum of 12+31+95 = 138. How many dates in 1995 have a date-sum of 130?

3. Mr. Jones finds an unflattering picture of himself on the chalkboard. When asked, George says, "Sally did it," Eric says, "George did it," Sally says, "George lied when he said I did it," and Julie says, "I didn't do it." Only one of the four is telling the truth and one of them did draw the picture.
   a) Who is telling the truth?
   b) Who drew the picture?

ANSWERS

(1 pt) 1._____

(2 pts.) 2._____

(3 pts.) 3.a_____ b._____

Hudson, St. John's, Tahanto, Tantasqua

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ROUND IV: Sequences and Series

ANSWERS MUST BE IN SIMPLEST EXACT FORM

1. If an ordinary striking clock were turned into a 24-hour clock, so that at midnight it struck 24 times, how many times would the clock strike in a full 24 hour day?

2. If $\sum_{j=1}^{5} (jk + 2) = 20$, find k.

3. The second term in a geometric sequence, which is less than its first term, is 12. If 3 is subtracted from the first term, and 1 is added to the third term, the three terms form an arithmetic sequence. Find the sum of the three terms of the geometric sequence.

ANSWERS

(1 pt) 1._____

(2 pts.) 2._____

(3 pts.) 3._____

Clinton, Quaboag, Tahanto

ROUND V: Matrix and determinant operations

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO 3 PLACES AFTER THE DECIMAL POINT

1. Write this system of equations as one equation involving three matrices, one of which is: $\begin{bmatrix} x \\ y \end{bmatrix}$ DO NOT SOLVE!

$$\begin{cases} 4x - 12y = 7 \\ x + 6y = 9 \end{cases}$$

2. Multiply Completely:

$$\left(\begin{bmatrix} 2 & -3 \\ 1 & 5 \\ -2 & 4 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 \\ -2 & -3 \end{bmatrix} \right) \cdot \begin{bmatrix} 3 \\ -2 \end{bmatrix}$$

3. $\begin{bmatrix} a & b \\ b & b \end{bmatrix} \cdot \begin{bmatrix} b & a \\ a & b \end{bmatrix} = \begin{bmatrix} d & d \\ d & d \end{bmatrix}$ and $\begin{vmatrix} a & 0 \\ 0 & b \end{vmatrix} = 8$

Find a if $a > 0$.

ANSWERS

(1 pt) 1._____

(2 pts.) 2. $\begin{bmatrix} & \\ & \end{bmatrix}$

(3 pts.) 3._____

Algonquin, Bartlett, Tahanto

TEAM ROUND : Topics of previous rounds and open

ALL ANSWERS MUST BE IN SIMPLEST EXACT FORM OR AS DECIMALS ROUNDED TO 3 PLACES AFTER THE DECIMAL POINT AND ON THE TEAM ANSWER SHEET. 2 POINTS EACH

1. What fraction greater than $\frac{29}{50}$ and less than $\frac{3}{5}$ has the smallest denominator?

2. $\frac{11}{13} = \frac{1}{A} + \frac{1}{B} + \frac{1}{C}$ for positive integers A, B, C. Evaluate $A + B + C$

3. Four cards have one side visible, showing A, B, 3, & 6, one on each card. Each card has a letter on one side and a number on the other. Specifying it or them by what is visible , which card(s) must be turned over to test whether this statement is correct? "If a card has A on one side, then it has 3 on the other."

4. Compute $1^2 - 2^2 + 3^2 - 4^2 + \ldots - 1998^2 + 1999^2$

5. Evaluate this determinant in terms of a and b :

$$\begin{vmatrix} a-b & 1 & 0 \\ b-a & 0 & a \\ a-b & 1 & b-a \end{vmatrix}$$

6. Find the area of a triangle whose vertices are (1,2), (6, -4), and (-2, -2).

7. What is half of 4^{x+3} ? Express the result as a power without fractions.

8. Let P(n) = the number of distinct prime factors in the integer n .
Evaluate P(P(P(210))).

9. Quadrilateral QUAD has $\angle Q : \angle U : \angle A : \angle D = 1 : 2 : 3 : 4$. $\overline{QD}$ and $\overline{UA}$ are extended to meet at point X forming ΔQXU. Find the degree measure of $\angle X$

Algonquin, Burncoat, Holy Name, Hudson, Leicester, Mass Academy, QSC,
St. John's, Worcester Academy

December 6, 1995 WOCOMAL VARSITY MEET ANSWERS

Round I Arith

1. 1 pt 39
2. 2 pts $\frac{1}{35}$
3. 3 pts $16\frac{2}{3}\%$

Round II Alg 1

1. 1 pt 38°
2. 2 pts 49
3. 3 pts -11

Round III sets logic

1. 1 pt 2
2. 2 pts 8 (need both)
3. 3 pts a) Sally b) Julie

Round IV seq series

1. 1 pt 300
2. 2 pts $\frac{2}{3}$
3. 3 pts 38

Round V matrix det

1. 1 pt $\begin{bmatrix} 4 & -12 \\ 1 & 6 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 7 \\ 9 \end{bmatrix}$
2. 2 pts $\begin{bmatrix} -2 \\ -1 \\ 2 \end{bmatrix}$
3. 3 pts $2\sqrt{2}$ or 2.828

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alt 1. $\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & -12 \\ 1 & 6 \end{bmatrix}^{-1}\begin{bmatrix} 7 \\ 9 \end{bmatrix}$

TEAM ROUND 2 pts each

1. $\frac{7}{12}$
2. 83
3. A and 6 (need both)
4. 1,999,000
5. $-(b-a)^2$ or equivalent
6. 19
7. 2^{2x+5}
8. 0
9. 72°